

Properties of Logarithms

Let b , m , and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Product Property:

Expand: $\log 15x$ $\log_3 2y$
 $\log 15 + \log x$ $\log_3 2 + \log_3 y$

Condense: $\log_3 4 + \log_3 y$ $\ln 2 + \ln x$
 $\log_3 4y$ $\ln 2x$

Quotient Property:

Expand: $\ln \frac{12}{5}$ $\log_5 \frac{x}{3}$
 $\ln 12 - \ln 5$ $\log_5 x - \log_5 3$

Condense: $\log_3 4 - \log_3 y$ $\ln x - \ln 3$
 $\log_3 \left(\frac{4}{y}\right)$ $\ln \frac{x}{3}$

Expand: $\log_4 \frac{3}{2x} = \log_4 3 - \log_4 2x$
 $\log_4 3 - (\log_4 2 + \log_4 x)$
 $\log_4 3 - \log_4 2 - \log_4 x$

Condense: $\log_5 3 + \log_5 x - \log_5 2$
 $\log_5 \frac{3x}{2}$

Power Property:

$$\log_2 m^5 = 5 \log_2 m$$

Expand: $\log x^4 = 4 \log x$

$\log_3 2x^4$
 $\log_3 2 + \log_3 x^4$
 $\log_3 2 + 4 \log_3 x$

$\ln 8x^3$
 $\ln 8 + \ln x^3$
 $\ln 8 + 3 \ln x$

Condense:

$\ln 12 - \ln 4$
 $\ln \left(\frac{12}{4}\right) = \ln 3$

$6 \ln 2 - 4 \ln y$
 $\ln 2^6 - \ln y^4$
 $\ln \left(\frac{2^6}{y^4}\right)$

$\log_3 \sqrt{y}$
 $\log_3 y^{\frac{1}{2}}$
 $\frac{1}{2} \log_3 y$
 $5 \log x - 4 \log y$
 $\log x^5 - \log y^4$
 $\log \left(\frac{x^5}{y^4}\right)$